Presented by:
Steven Kao
Michelle Maestas

Calculus 3 Prep
(Calc 2 Review)

These interactive workshops will review all foundational material leading up to the specified course so you are better equipped to hit the ground running.

Synchronous in-person in the ESS suite & virtual via Zoom

<table>
<thead>
<tr>
<th>Workshop</th>
<th>Date</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physics 1 Prep</td>
<td>Monday, January 10, 2022</td>
<td>10 AM - 12 PM</td>
</tr>
<tr>
<td>Pre-Calc/Trip Prep (College Algebra Review)</td>
<td>Tuesday, January 11, 2022</td>
<td>10 AM - 12 PM</td>
</tr>
<tr>
<td>Calc 1 Prep (Pre-Calc/Trip Review) - PM offering</td>
<td>Tuesday, January 11, 2022</td>
<td>1 - 3 PM</td>
</tr>
<tr>
<td>Chem 1 Prep</td>
<td>Wednesday, January 12, 2022</td>
<td>10 AM - 12 PM</td>
</tr>
<tr>
<td>Calc 1 Prep (Pre-Calc/Trip Review) - AM offering</td>
<td>Thursday, January 13, 2022</td>
<td>10 AM - 12 PM</td>
</tr>
<tr>
<td>Calc 2 Prep (Calc 1 Review)</td>
<td>Thursday, January 13, 2022</td>
<td>1 - 3 PM</td>
</tr>
<tr>
<td>Calc 3 Prep (Calc 2 Review)</td>
<td>Friday, January 14, 2022</td>
<td>10 AM - 12 PM</td>
</tr>
</tbody>
</table>

RSVP is preferred but not required
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Calc II Review

Important Concepts:
- Inverse function theorem and its applications
- Integration techniques: integration by parts, trig reduction, trig substitution, partial fraction decomposition
- Improper integrals
- Convergence tests for sequences and series
Inverse Functions

Let \( f(x) \) be a 1-to-1 function having domain \( A \) and range \( B \).

The inverse function \( f^{-1}(x) \) satisfies the following “cancellation” properties:

1. \( f^{-1}(f(x)) = x \) for every \( x \in A \).
2. \( f(f^{-1}(y)) = y \) for every \( y \in B \).

Conversely, any function \( f^{-1}(x) \) satisfying the above properties is the inverse of \( f(x) \).

Key Points:

• A function MUST be 1-to-1 in order to have an inverse.
• The inverse function “undoes” the original function.
• This means the domain of the inverse function \( f^{-1}(x) \) is the range \( B \) of the original function \( f(x) \), and the range of \( f^{-1}(x) \) is the domain \( A \) of \( f(x) \).
Inverse function theorem

Let $f$ be a differentiable function having an inverse on the interval $I$. If $x_0$ is a point of $I$ such that $f'(x_0) \neq 0$, then $f^{-1}$ is differentiable at $y_0 = f(x_0)$ and

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)}, \quad \text{where} \quad y_0 = f(x_0)$$
Find the derivative of $f^{-1}(y_0)$ given the function $f$ and the point $y_0$:

1. $f(x) = \sqrt{x} + x^2 + 2$, $y_0 = 4$
2. $f(x) = \tan(\pi x)$, $y_0 = 1$
3. $f(x) = \frac{x}{x-6}$, $y_0 = 4$
Integration Techniques
Integration by Parts

Given two differentiable functions $u$ and $v$, the Product Rule states that

$$\frac{d}{dx} (u(x)v(x)) = u'(x)v(x) + u(x)v'(x).$$

By integrating both sides, we can write this rule in terms of an indefinite integral:

$$u(x)v(x) = \int (u'(x)v(x) + u(x)v'(x)) \, dx.$$  

Rearranging this expression in the form

$$\int u(x)v'(x) \, dx = u(x)v(x) - \int v(x)u'(x) \, dx$$

### Integration by Parts

Suppose $u$ and $v$ are differentiable functions. Then

$$\int u \, dv = uv - \int v \, du.$$
Example

Evaluate the integral.

\[ \int x^2 2^x \, dx \]
Practice

1) $\int \frac{\ln x}{x^{10}} \, dx$

2) $\int x^2 (\ln x)^2 \, dx$

3) $\int \sin^{-1} x \, dx$

4) $\int e^{\sqrt{x}} \, dx$

5) $\int (\sec x)^3 \, dx$
Trig Integrals

<table>
<thead>
<tr>
<th>Integral</th>
<th>Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\int \sin^m x \cos^n x , dx$</td>
<td>Split off $\sin x$, rewrite the resulting even power of $\sin x$ in terms of $\cos x$, and then use $u = \cos x$.</td>
</tr>
<tr>
<td>$m$ odd and positive, $n$ real</td>
<td></td>
</tr>
<tr>
<td>$n$ odd and positive, $m$ real</td>
<td>Split off $\cos x$, rewrite the resulting even power of $\cos x$ in terms of $\sin x$, and then use $u = \sin x$.</td>
</tr>
<tr>
<td>$m$ and $n$ both even, nonnegative integers</td>
<td>Use half-angle formulas to transform the integrand into a polynomial in $\cos 2x$, and apply the preceding strategies once again to powers of $\cos 2x$ greater than 1.</td>
</tr>
<tr>
<td>$\int \tan^m x \sec^n x , dx$</td>
<td>Split off $\sec^2 x$, rewrite the remaining even power of $\sec x$ in terms of $\tan x$, and use $u = \tan x$.</td>
</tr>
<tr>
<td>$n$ even and positive, $m$ real</td>
<td></td>
</tr>
<tr>
<td>$m$ odd and positive, $n$ real</td>
<td>Split off $\sec x \tan x$, rewrite the remaining even power of $\tan x$ in terms of $\sec x$, and use $u = \sec x$.</td>
</tr>
</tbody>
</table>
Example

Evaluate the integral.

\[ \int (\sin x)^2 (\cos x)^2 \, dx \]
Practice

25. \[ \int \sin^2 x \cos^4 x \, dx \]
26. \[ \int \sin^3 x \cos^{3/2} x \, dx \]
27. \[ \int \tan^2 x \, dx \]
28. \[ \int 6 \sec^4 x \, dx \]
35. \[ \int \tan x \sec^3 x \, dx \]
36. \[ \int \tan 4x \sec^{3/2} 4x \, dx \]
Trig Substitution

The Integral Contains . . .

\[ a^2 - x^2 \quad x = a \sin \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \text{ for } |x| \leq a \]

\[ a^2 + x^2 \quad x = a \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \]

\[ x^2 - a^2 \quad x = a \sec \theta, \quad \begin{cases} 
0 \leq \theta < \frac{\pi}{2}, \text{ for } x \geq a \\
\frac{\pi}{2} < \theta \leq \pi, \text{ for } x \leq -a
\end{cases} \]

Useful Identity

\[ a^2 - a^2 \sin^2 \theta = a^2 \cos^2 \theta \]

\[ a^2 + a^2 \tan^2 \theta = a^2 \sec^2 \theta \]

\[ a^2 \sec^2 \theta - a^2 = a^2 \tan^2 \theta \]
Example

Evaluate the integral.

\[ \int \frac{\sqrt{x^2+4x-5}}{x+2} \, dx \]
Practice

18. \[ \int \frac{dx}{(1 + x^2)^{3/2}} \]

19. \[ \int \frac{dx}{\sqrt{x^2 - 81}}, x > 9 \]

20. \[ \int \frac{dx}{\sqrt{x^2 - 49}}, x > 7 \]

21. \[ \int \sqrt{64 - x^2} \, dx \]

22. \[ \int \frac{dt}{t^2 \sqrt{9 - t^2}} \]

23. \[ \int \frac{dx}{(25 - x^2)^{3/2}} \]
### Partial Fractions

#### SUMMARY  Partial Fraction Decompositions

Let \( f(x) = \frac{p(x)}{q(x)} \) be a proper rational function in reduced form. Assume the denominator \( q \) has been factored completely over the real numbers and \( m \) is a positive integer.

1. **Simple linear factor** A factor \( x - r \) in the denominator requires the partial fraction \( \frac{A}{x - r} \).

2. **Repeated linear factor** A factor \((x - r)^m\) with \( m > 1 \) in the denominator requires the partial fractions
   \[
   \frac{A_1}{(x - r)} + \frac{A_2}{(x - r)^2} + \frac{A_3}{(x - r)^3} + \cdots + \frac{A_m}{(x - r)^m}.
   \]

3. **Simple irreducible quadratic factor** An irreducible factor \( ax^2 + bx + c \) in the denominator requires the partial fraction
   \[
   \frac{Ax + B}{ax^2 + bx + c}.
   \]

4. **Repeated irreducible quadratic factor** (See Exercises 61–64.) An irreducible factor \((ax^2 + bx + c)^m\) with \( m > 1 \) in the denominator requires the partial fractions
   \[
   \frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_mx + B_m}{(ax^2 + bx + c)^m}.
   \]
Example

Evaluate the integral.

\[ \int \frac{10}{(x-2)^2(x^2+2x+2)} \, dx \]
Practice

45. \[ \int \frac{x - 5}{x^2(x + 1)} \, dx \]

46. \[ \int \frac{x^2}{(x - 2)^3} \, dx \]

47. \[ \int \frac{x^3 - 10x^2 + 27x}{x^2 - 10x + 25} \, dx \]

48. \[ \int \frac{x^3 + 2}{x^3 - 2x^2 + x} \, dx \]

49. \[ \int \frac{x^2 - 4}{x^3 - 2x^2 + x} \, dx \]

50. \[ \int \frac{8(x^2 + 4)}{x(x^2 + 8)} \, dx \]
Improper Integrals, Part I

**DEFINITION** Improper Integrals over Infinite Intervals

1. If \( f \) is continuous on \([a, \infty)\), then
   \[
   \int_a^\infty f(x) \, dx = \lim_{b \to \infty} \int_a^b f(x) \, dx.
   \]

2. If \( f \) is continuous on \((\infty, b]\), then
   \[
   \int_{-\infty}^b f(x) \, dx = \lim_{a \to -\infty} \int_a^b f(x) \, dx.
   \]

3. If \( f \) is continuous on \((\infty, \infty)\), then
   \[
   \int_{-\infty}^{\infty} f(x) \, dx = \lim_{a \to -\infty} \int_a^c f(x) \, dx + \lim_{b \to \infty} \int_c^b f(x) \, dx,
   \]
   where \( c \) is any real number. It can be shown that the choice of \( c \) does not affect the value or convergence of the original integral.

   If the limits in cases 1–3 exist, then the improper integrals **converge**; otherwise, they **diverge**.
Improper Integrals, Part II

**DEFINITION** Improper Integrals with an Unbounded Integrand

1. Suppose \( f \) is continuous on \((a, b] \) with 
\[
\lim_{x \to b^-} f(x) = \pm \infty.
\]
Then
\[
\int_a^b f(x) \, dx = \lim_{c \to b^-} \int_c^b f(x) \, dx.
\]

2. Suppose \( f \) is continuous on \([a, b) \) with 
\[
\lim_{x \to a^+} f(x) = \pm \infty.
\]
Then
\[
\int_a^b f(x) \, dx = \lim_{c \to a^+} \int_a^c f(x) \, dx.
\]

3. Suppose \( f \) is continuous on \([a, b) \) except at the interior point \( c \) where \( f \) is unbounded. Then
\[
\int_a^b f(x) \, dx = \lim_{c \to c^-} \int_a^c f(x) \, dx + \lim_{d \to c^+} \int_c^d f(x) \, dx.
\]
If the limits in cases 1–3 exist, then the improper integrals converge; otherwise, they diverge.
Example

Evaluate the integral.

\[ \int_{0}^{3} \frac{1}{x-1} \, dx \]
Practice

35. \( \int_{-\infty}^{0} \frac{dx}{\sqrt[3]{2 - x}} \)

37. \( \int_{0}^{8} \frac{dx}{\sqrt[3]{x}} \)

39. \( \int_{0}^{\pi/2} \tan \theta \, d\theta \)

36. \( \int_{e^2}^{\infty} \frac{dx}{x \ln^p x}, p > 1 \)

38. \( \int_{1}^{2} \frac{dx}{\sqrt{x - 1}} \)

40. \( \int_{-3}^{1} \frac{dx}{(2x + 6)^{2/3}} \)
Sequences and Series
Sequences

**DEFINITION**  Limit of a Sequence

If the terms of a sequence \( \{a_n\} \) approach a unique number \( L \) as \( n \) increases—that is, if \( a_n \) can be made arbitrarily close to \( L \) by taking \( n \) sufficiently large—then we say \( \lim_{n \to \infty} a_n = L \) exists, and the sequence **converges** to \( L \). If the terms of the sequence do not approach a single number as \( n \) increases, the sequence has no limit, and the sequence **diverges**.
Properties of Sequences

**Theorem 10.2** Limit Laws for Sequences
Assume the sequences \( \{a_n\} \) and \( \{b_n\} \) have limits A and B, respectively. Then

1. \( \lim_{n \to \infty} (a_n \pm b_n) = A \pm B \)

2. \( \lim_{n \to \infty} c a_n = cA \), where \( c \) is a real number

3. \( \lim_{n \to \infty} a_n b_n = AB \)

4. \( \lim_{n \to \infty} \frac{a_n}{b_n} = \frac{A}{B} \), provided \( B \neq 0 \).

**Theorem 10.4** Squeeze Theorem for Sequences
Let \( \{a_n\} \), \( \{b_n\} \), and \( \{c_n\} \) be sequences with \( a_n \leq b_n \leq c_n \) for all integers \( n \) greater than some index \( N \). If \( \lim_{n \to \infty} a_n = \lim_{n \to \infty} c_n = L \), then \( \lim_{n \to \infty} b_n = L \) (Figure 10.19).

**Theorem 10.5** Bounded Monotonic Sequence
A bounded monotonic sequence converges.
Examples

a. \[ \left\{ \frac{(-1)^n}{n^2 + 1} \right\}_{n=1}^{\infty} \]

b. \[ \{ \cos n\pi \}_{n=1}^{\infty} \]

c. \[ \{ a_n \}_{n=1}^{\infty}, \text{ where } a_{n+1} = 2a_n, a_1 = 1 \]
Infinite Series

DEFINITION Infinite Series
Given a sequence \( \{a_1, a_2, a_3, \ldots \} \), the sum of its terms
\[
a_1 + a_2 + a_3 + \cdots = \sum_{k=1}^{\infty} a_k
\]
is called an infinite series. The sequence of partial sums \( \{S_n\} \) associated with this series has the terms
\[
\begin{align*}
S_1 &= a_1 \\
S_2 &= a_1 + a_2 \\
S_3 &= a_1 + a_2 + a_3 \\
\vdots \\
S_n &= a_1 + a_2 + a_3 + \cdots + a_n = \sum_{k=1}^{n} a_k, \quad \text{for } n = 1, 2, 3, \ldots
\end{align*}
\]

If the sequence of partial sums \( \{S_n\} \) has a limit \( L \), the infinite series converges to that limit, and we write
\[
\sum_{k=1}^{\infty} a_k = \lim_{n \to \infty} \sum_{k=1}^{n} a_k = \lim_{n \to \infty} S_n = L.
\]
If the sequence of partial sums diverges, the infinite series also diverges.
THEOREM 10.8  Properties of Convergent Series

1. Suppose \( \sum a_k \) converges to \( A \) and \( c \) is a real number. The series \( \sum ca_k \) converges, and \( \sum ca_k = c\sum a_k = cA \).

2. Suppose \( \sum a_k \) diverges. Then \( \sum ca_k \) also diverges, for any real number \( c \neq 0 \).

3. Suppose \( \sum a_k \) converges to \( A \) and \( \sum b_k \) converges to \( B \). The series \( \sum (a_k \pm b_k) \) converges, and \( \sum (a_k \pm b_k) = \sum a_k \pm \sum b_k = A \pm B \).

4. Suppose \( \sum a_k \) diverges and \( \sum b_k \) converges. Then \( \sum (a_k \pm b_k) \) diverges.

5. If \( M \) is a positive integer, then \( \sum_{k=1}^\infty a_k \) and \( \sum_{k=M}^\infty a_k \) either both converge or both diverge. In general, whether a series converges does not depend on a finite number of terms added to or removed from the series. However, the value of a convergent series does change if nonzero terms are added or removed.
THEOREM 10.7 Geometric Series

Let $a \neq 0$ and $r$ be real numbers. If $|r| < 1$, then

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1 - r}.$$  

If $|r| \geq 1$, then the series diverges.

- Diverges $r \leq -1$
- Converges $-1 < r < 1$
- Diverges $r \geq 1$
Telescoping Series

\[
\sum_{k=1}^{\infty} \left( \frac{\cos \frac{1}{k}}{k} - \frac{\cos \frac{1}{k+1}}{k+1} \right)
\]

\[
S_n = \sum_{k=1}^{n} \left( \frac{\cos \frac{1}{k}}{k} - \frac{\cos \frac{1}{k+1}}{k+1} \right)
\]

\[
= \left( \frac{\cos 1}{1} - \frac{\cos \frac{1}{2}}{2} \right) + \left( \frac{\cos \frac{1}{2}}{2} - \frac{\cos \frac{1}{3}}{3} \right) + \cdots + \left( \frac{\cos \frac{1}{n}}{n} - \frac{\cos \frac{1}{n+1}}{n+1} \right)
\]

\[
= \cos 1 + \left( -\cos \frac{1}{2} + \cos \frac{1}{2} \right) + \cdots + \left( -\cos \frac{1}{n} + \cos \frac{1}{n} \right) - \cos \frac{1}{n+1}
\]

\[
= \cos 1 - \cos \frac{1}{n+1}.
\]

Regroup terms.

Simplify.

\[
\sum_{k=1}^{\infty} \left( \frac{\cos \frac{1}{k}}{k} - \frac{\cos \frac{1}{k+1}}{k+1} \right) = \lim_{n \to \infty} S_n
\]

\[
= \lim_{n \to \infty} \left( \cos 1 - \cos \frac{1}{n+1} \right) = \cos 1 - 1
\]

\[
\rightarrow \cos 0 = 1
\]
Practice

74. \[ \sum_{k=0}^{\infty} \left( \sin \left( \frac{(k + 1)\pi}{2k + 1} \right) - \sin \left( \frac{k\pi}{2k - 1} \right) \right) \]

75. \[ \sum_{k=0}^{\infty} \left( \frac{1}{4} \right)^{5^{3-k}} \]

79. \[ \sum_{k=2}^{\infty} \frac{\ln \left( (k + 1)k^{-1} \right)}{(\ln k)\ln (k + 1)} \]

81. \[ \sum_{k=0}^{\infty} \left( 3 \left( \frac{2}{5} \right)^k - 2 \left( \frac{5}{7} \right)^k \right) \]

76. \[ \sum_{k=2}^{\infty} \left( \frac{3}{8} \right)^{3k} \]

77. \[ \sum_{k=2}^{\infty} \left( \frac{3}{8} \right)^{3k} \]

80. \[ \sum_{k=1}^{\infty} \frac{\pi^k}{e^{k+1}} \]

82. \[ \sum_{k=1}^{\infty} \left( 2 \left( \frac{3}{5} \right)^k + 3 \left( \frac{4}{9} \right)^k \right) \]
Divergence Test

**THEOREM 10.9  Divergence Test**

If $\sum a_k$ converges, then $\lim_{k \to \infty} a_k = 0$. Equivalently, if $\lim_{k \to \infty} a_k \neq 0$, then the series diverges.
THEOREM 10.11  Integral Test
Suppose $f$ is a continuous, positive, decreasing function, for $x \geq 1$, and let $a_k = f(k)$, for $k = 1, 2, 3, \ldots$. Then

$$\sum_{k=1}^{\infty} a_k \quad \text{and} \quad \int_{1}^{\infty} f(x) \, dx$$

either both converge or both diverge. In the case of convergence, the value of the integral is not equal to the value of the series.
Consequence of the Integral Test

**THEOREM 10.12  Convergence of the $p$-series**

The $p$-series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges for $p > 1$ and diverges for $p \leq 1$. 
Comparison Test

**THEOREM 10.14  Comparison Test**

Let $\sum a_k$ and $\sum b_k$ be series with positive terms.

1. If $a_k \leq b_k$ and $\sum b_k$ converges, then $\sum a_k$ converges.
2. If $b_k \leq a_k$ and $\sum b_k$ diverges, then $\sum a_k$ diverges.
**Theorem 10.15  Limit Comparison Test**

Let $\sum a_k$ and $\sum b_k$ be series with positive terms and let

$$\lim_{k \to \infty} \frac{a_k}{b_k} = L.$$

1. If $0 < L < \infty$ (that is, $L$ is a finite positive number), then $\sum a_k$ and $\sum b_k$ either both converge or both diverge.
2. If $L = 0$ and $\sum b_k$ converges, then $\sum a_k$ converges.
3. If $L = \infty$ and $\sum b_k$ diverges, then $\sum a_k$ diverges.
**THEOREM 10.16** Alternating Series Test
The alternating series \( \sum (-1)^{k+1} a_k \) converges provided

1. the terms of the series are nonincreasing in magnitude (\( 0 < a_{k+1} \leq a_k \), for \( k \) greater than some index \( N \)) and
2. \( \lim_{k \to \infty} a_k = 0 \).
Consequence of Alternating Series Test

**THEOREM 10.17 Alternating Harmonic Series**

The alternating harmonic series \( \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \) converges (even though the harmonic series \( \sum_{k=1}^{\infty} \frac{1}{k} \) diverges).
Absolute Convergence Test

**THEOREM 10.19 Absolute Convergence Implies Convergence**

If $\sum |a_k|$ converges, then $\sum a_k$ converges (absolute convergence implies convergence). Equivalently, if $\sum a_k$ diverges, then $\sum |a_k|$ diverges.
Absolute vs. Conditional Convergence

**DEFINITION** Absolute and Conditional Convergence

If $\sum |a_k|$ converges, then $\sum a_k$ converges absolutely.

If $\sum |a_k|$ diverges and $\sum a_k$ converges, then $\sum a_k$ converges conditionally.
Ratio Test

**THEOREM 10.20  Ratio Test**

Let $\sum a_k$ be an infinite series, and let $r = \lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right|$. 

1. If $r < 1$, the series converges absolutely, and therefore it converges (by Theorem 10.19).
2. If $r > 1$ (including $r = \infty$), the series diverges.
3. If $r = 1$, the test is inconclusive.
Root Test

**Theorem 10.21** Root Test
Let $\sum a_k$ be an infinite series, and let $\rho = \lim_{k \to \infty} \sqrt[k]{|a_k|}$.

1. If $\rho < 1$, the series converges absolutely, and therefore it converges (by Theorem 10.19).
2. If $\rho > 1$ (including $\rho = \infty$), the series diverges.
3. If $\rho = 1$, the test is inconclusive.
Practice

23. \( \sum_{k=1}^{\infty} \frac{k^5}{5^k} \)

25. \( \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}e\sqrt{k}} \)

27. \( \sum_{k=1}^{\infty} \frac{3 + \cos 5k}{k^3} \)

29. \( \sum_{k=1}^{\infty} \frac{10^k + 1}{k^{10}} \)

31. \( \sum_{j=1}^{\infty} \frac{5}{j^2 + 4} \)

33. \( \sum_{k=1}^{\infty} \frac{1}{k^{1/3} \ln k} \)

35. \( \sum_{k=1}^{\infty} \frac{2k^3}{k^k} \)

24. \( \sum_{k=1}^{\infty} \frac{4}{(k + 3)^3} \)

26. \( \sum_{k=1}^{\infty} \frac{5 + \sin k}{\sqrt{k}} \)

28. \( \sum_{k=3}^{\infty} \frac{(-1)^k \ln k}{k^{1/3}} \)

30. \( \sum_{k=3}^{\infty} \frac{1}{k^3 \ln k} \)

32. \( \sum_{k=1}^{\infty} \frac{k^k}{(k + 2)^k} \)

34. \( \sum_{k=1}^{\infty} \frac{(-1)^k 5k^2}{\sqrt{3k^3 + 1}} \)

36. \( \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2k + 1}{k!} \)
Thank You!

Questions?