Calculus 3 Prep Problems

Derivatives

Use the specified rule to find the derivative of each of the following problems:

Power:

$$1.\frac{d}{dx}5x^{20}$$

Product:

$$2.\frac{d}{dx}e^x\sin(x)$$

Quotient:

$$3.\frac{d}{dx}\frac{2x}{\sin(x)}$$

Chain:

$$4.\frac{d}{dx} 5(x-3)^2$$

Use the inverse function theorem, as state below, to find the derivative of the inverse of each of the following functions.

$$(f^{-1})'(x) = \left[f'(f^{-1}(x))\right]^{-1} = \frac{1}{f'(f^{-1}(x))}$$

Find $(f^{-1})'(x)$
5. $f(x) = e^x$

6. $f(x) = 2x^2 + 4$

 $7.f(x) = \sin(x)$

Optimization

8. Find two positive numbers whose sum is 50 and whose product is a maximum.

9. We have 45 m² of material to build a box with a square base and no top. Determine the dimensions of the box that will maximize the enclosed volume.

10. A car rental company charges its customers x dollars per day, where $60 \le x \le 150$. It has found that the number of cars rented per day can be modeled by the linear function n(x) = 750 - 5x. How much should the company charge each customer to maximize revenue? Graphing

11. Which of the below graphs represents the derivative of f(x)?



12. Which of the below graphs represents the function f(x)?



13. If the function f(x) is represented by the graph below, sketch a graph of f'(x):



Integration

Find the integral of the following functions using the specified method:

U-Substitution

Recall rule

$$\int f(g(x))g'(x)dx = \int f(u)du, \quad u = g(x), \quad du = g'(x)$$
14.
$$\int 2x(x^2 + 4)^3 dx$$

$$15.\int \sin(x)\cos^2(x)\,dx$$

$$16. \int \frac{2\ln x}{x} dx$$

Integration by parts

Recall rule

$$\int u dv = uv - \int v du$$

$$17. \int x \sin(x) dx$$

$$18.\int x^2 e^x dx$$

$19.\int x^{-3}\ln x\,dx$

Improper integrals

$$20.\int_0^\infty x^2 dx$$

$$21.\int_{1}^{\infty}\frac{1}{x^2}$$

$$22. \int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$$