## Calculus 3 Prep Problems

## Derivatives

Use the specified rule to find the derivative of each of the following problems:
Power:

1. $\frac{d}{d x} 5 x^{20}$

Product:
2. $\frac{d}{d x} e^{x} \sin (x)$

Quotient:
3. $\frac{d}{d x} \frac{2 x}{\sin (x)}$

Chain:
4. $\frac{d}{d x} 5(x-3)^{2}$

Use the inverse function theorem, as state below, to find the derivative of the inverse of each of the following functions.

$$
\left(f^{-1}\right)^{\prime}(x)=\left[f^{\prime}\left(f^{-1}(x)\right)\right]^{-1}=\frac{1}{f^{\prime}\left(f^{-1}(x)\right)}
$$

Find $\left(f^{-1}\right)^{\prime}(x)$
5. $f(x)=e^{x}$
6. $f(x)=2 x^{2}+4$
7. $f(x)=\sin (x)$
8. Find two positive numbers whose sum is 50 and whose product is a maximum.
9. We have $45 \mathrm{~m}^{2}$ of material to build a box with a square base and no top. Determine the dimensions of the box that will maximize the enclosed volume.
10. A car rental company charges its customers $x$ dollars per day, where $60 \leq x \leq 150$. It has found that the number of cars rented per day can be modeled by the linear function $n(x)=750-5 x$. How much should the company charge each customer to maximize revenue?
11. Which of the below graphs represents the derivative of $f(x)$ ?


b

12. Which of the below graphs represents the function $f(x)$ ?




13. If the function $f(x)$ is represented by the graph below, sketch a graph of $f^{\prime}(x)$ :


## Integration

Find the integral of the following functions using the specified method:
U-Substitution

Recall rule
$\int f(g(x)) g^{\prime}(x) d x=\int f(u) d u, \quad u=g(x), \quad d u=g^{\prime}(x)$
14. $\int 2 x\left(x^{2}+4\right)^{3} d x$
15. $\int \sin (x) \cos ^{2}(x) d x$
16. $\int \frac{2 \ln x}{x} d x$

Integration by parts

## Recall rule

$\int u d v=u v-\int v d u$
17. $\int x \sin (x) d x$
18. $\int x^{2} e^{x} d x$
19. $\int x^{-3} \ln x d x$

Improper integrals
20. $\int_{0}^{\infty} x^{2} d x$
21. $\int_{1}^{\infty} \frac{1}{x^{2}}$
22. $\int_{-\infty}^{\infty} \frac{1}{1+x^{2}} d x$

