Calculus 2 Prep August 2023

# Calculus 2 Prep Problems

# Limits

Use L'Hopital's Rule to solve the following problems:

$$1. \lim_{x \to 0} \frac{\sin(x)}{x}$$

$$2. \lim_{x \to \infty} \frac{4x^2 + 12}{e^x}$$

### Continuity

Find all asymptotes and holes in the following functions. State the interval(s) on which each function is continuous.

3. 
$$f(x) = |2x + 2|$$

$$4. h(x) = e^{-x}$$

$$5. g(x) = \frac{x}{x^2 - 25}$$

# Derivatives

Use the indicated rule to derive each of the following functions.

Power Rule

$$6.\frac{d}{dx}5x^{20}$$

**Product Rule** 

$$7.\frac{d}{dx}e^x\sin(x)$$

**Quotient Rule** 

$$8.\frac{d}{dx}\frac{2x}{\sin(x)}$$

Chain Rule

$$9.\frac{d}{dx} 5(x-3)^2$$

$$10.\frac{d}{dx}\sin(\ln(x))$$

Identifying Derivative Rules

Which rule(s) do you need to solve these? Then take the derivative.

$$11.\frac{d}{dx}\frac{3}{x^3}$$

$$12.\frac{d}{dx}\frac{\sin(x)\cos(x)}{2x}$$

$$13.\frac{d}{dx}2x\sin(x)\cos(x)$$

$$14.\frac{d}{dx} \left[ \frac{1}{\sin(x)} + \frac{2}{\sin^2(x)} \right]$$

#### Extrema

Solve f'(x) = 0 to find critical points and identify the maximum and minimum of the function:

$$15. f(x) = x^3 + 3x^2 + x$$

### Concavity

Solve f''(x) = 0 to find inflection points, and identify areas of concavity.

$$16. f(x) = \frac{1}{12}x^4 - \frac{7}{6}x^3 - 9x^2 + 12x + 5$$

# Graphing using derivatives

Sketch the graph for the given functions by using properties of derivatives

$$17. f(x) = x^3 + 3x^2 + x$$

$$18. f(x) = \frac{1}{12}x^4 - \frac{7}{6}x^3 - 9x^2 + 12x + 5$$

# Integrals

Solve the following integrals by area — do not take the integral!

$$19. f(x) = \begin{cases} 5x, & x < 5 \\ 25, & 5 \le x < 10 \\ -5x + 75, & x \ge 10 \end{cases} \int_{0}^{15} f(x) dx$$

$$20. h(x) = \sqrt{36 - x^2}, \qquad \int_{-6}^{6} h(x) dx$$

Solve the following integrals with the specified rule.

Power Rule

$$21. \int 5x^7 + 3x^2 dx$$

$$22. \int x^{-5} + 3dx$$

U-substitution

$$23. \int 2x(x^2+4)^3 dx$$

$$24. \int \sin(x) \cos^2(x) \, dx$$

2<sup>nd</sup> Fundamental Theorem of Calculus

$$25. \int_0^{\pi} \sin(x) \, dx$$

$$26. \int_{-1}^{1} 2x^3 dx$$