Pre-Calc & Trig Review
(Calculus 1 Prep)

Presented by:
Natalie Burke
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Many are open to pre- and full majors and have no citizenship or GPA requirements.

MENTORING

- BE a mentor
  - to our incoming students in their transition into the University of New Mexico, the university setting, and Albuquerque.

- HAVE a mentor*
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*This program is open to UNM STEM Majors, Priority is given to Freshmen and Sophomores, but all levels are encouraged to apply.

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Includes 1-click RSVP
Precalc and Trig Review

Important Concepts:
- Ellipses and hyperbolas
- System of non-linear equations
- Unit circle, trig functions, and inverse trig functions
- Law of sine and cosine
- Trig identities and solving trig equations
Ellipses and Hyperbolas

Ellipses and hyperbolas are examples of conic sections, which have been studied since ancient times by the Greeks in 350-300 BCE by Menaechmus and Euclid.
Ellipses

Definition: An **ellipse** is the set of all points in the plane, the sum of whose distances from two fixed points $F_1$ and $F_2$ (called **foci**) is a constant, usually denoted as $2a$.

$$d_1 + d_2 = 2a$$
Parts of an ellipse

Ellipse with Horizontal Major Axis

- Vertex: (-a, 0)
- Focus: (-c, 0)
- Focus: (c, 0)
- Co-vertex: (0, b)
- Center: (0, 0)
- Minor axis
- Major axis

Ellipse with Vertical Major Axis

- Vertex: (0, -a)
- Focus: (0, -c)
- Focus: (0, c)
- Co-vertex: (b, 0)
- Center: (0, 0)
- Minor axis
- Major axis
Properties of ellipses

Because of the way ellipses are constructed, sound and light waves sent from one focus will reflect off the ellipse and bounce to the other focus.

Also, the distance from the origin to either focus, c, and the distance from the origin to the co-vertex, b, form the legs of a right triangle with hypotenuse equal to the distance from the origin to the vertex, a. So by Pythagorean Theorem:

\[ c^2 + b^2 = a^2 \Rightarrow c^2 = a^2 - b^2 \]
Equation of an ellipse

Eccentricity: \[ e = \frac{c}{a} \]

The eccentricity of an ellipse is always \( 0 < e < 1 \)
Practice

Graph and write the equation of the ellipse with the following properties:

1. Foci: \((±4, 0)\), vertices: \((±5, 0)\)
2. Foci: \((0, ±3)\), vertices: \((0, ±5)\)
3. Foci: \((2, 2)\) and \((0,2)\), vertices: \((3, 2)\) and \((-1, 2)\)
4. Foci: \((-2, -5)\) and \((-2, -1)\), vertices: \((-2, -7)\) and \((-2, 1)\)
Hyperbolas

Definition: A hyperbola is the set of all points in the plane, the difference of whose distances from two fixed points $F_1$ and $F_2$ (called foci) is a constant, usually denoted as $2a$.

$$|d_1 - d_2| = 2a$$
Parts of a hyperbola
Properties of Hyperbolas

Because of the way hyperbolas are constructed, sound and light waves sent towards one focus will reflect off the hyperbola and bounce to the other focus.

Also, the distance from the origin to the vertex, $a$, and $b$ form the legs of a right triangle with hypotenuse equal to the distance from the origin to the focus, $c$. So by Pythagorean Theorem:

\[ c^2 = a^2 + b^2 \]
Equation of a hyperbola

Eccentricity: \( e = \frac{c}{a} \)

The eccentricity of a hyperbola is always: \( e > 1 \)
Practice

Graph and write the equation of the hyperbola with the following properties:

1. Foci: $(\pm 5, 0)$, vertices: $(\pm 4, 0)$
2. Foci: $(0, \pm 5)$, vertices: $(0, \pm 3)$
3. Foci: $(3, 2)$ and $(-1, 2)$, vertices: $(2, 2)$ and $(0,2)$
4. Foci: $(-2, -7)$ and $(-2, 1)$, vertices: $(-2, -5)$ and $(-2, -1)$
Solving a system of nonlinear equations is no different than solving a system of linear equations. We can use either the method of substitution or elimination.
Example

\[ x^2 + y^2 = 100 \]
\[ 3x - y = 10 \]
Practice

1) \(3x^2 + 2y = 26\)
   \(5x^2 + 7y = 3\)

2) \(2x^2 - y^2 + 4 = 0\)
   \(xy = 24\)

3) \(\ln x + \ln y = \frac{3}{2}\)
   \(2 \ln x - \ln y = 0\)

4) \(2^x + 2^y = 10\)
   \(4^x + 4^y = 68\)
The Unit Circle
Inverse Trig Functions
Inverse Trig

1. An inverse trig function "undoes" the evaluation of the original trig function, so the input is now the ratio of two sides and the output is the angle.
2. Consequently, the domain and range of an inverse trig function will be the range and “restricted” domain of its corresponding trig function.
3. The domain of the original trig function has to be restricted so that it’s 1-to-1.

The three main inverse trig functions are:

- Inverse Sine: $\sin^{-1} x$ or $\arcsin x$
- Inverse Cosine: $\cos^{-1} x$ or $\arccos x$
- Inverse Tangent: $\tan^{-1} x$ or $\arctan x$
Inverse Trig Graphs
The Sine and Inverse Sine Graphs

Odd function
The Cosine and Inverse Cosine Graphs

Even function
The Tangent and Inverse Tangent Graphs

Odd function
Inverse Trig

Solve for $\theta$:

1. $\cos \theta = \frac{1}{2}$

2. $\sin \theta = \frac{\sqrt{2}}{2}$

3. $\theta = \sin^{-1} \left( -\frac{\sqrt{3}}{2} \right)$
Graphs of Trig Functions
Graphs of Trig Functions

The main points of a trig graph are:

- Period/Period Factor
- Frequency \((1/\text{Period})\)
- Phase Shift
- Horizontal Shift
- Vertical Shift
- Amplitude \((\sin x \text{ and } \cos x)\)
- Asymptotes \((\tan x)\)
- Oscillations
Graphs of Trig Functions

Let’s try graphing this together:

1.) \( f(x) = -3 \sin \left( \pi x + \frac{\pi}{2} \right) \)

2.) \( f(x) = |\sin x| \)

3.) \( f(x) = \frac{5 \cos(2x - \pi/2) + 20}{10} \)
Law of sines and cosines

- AAS, ASA: Law of Sines
- SSA: Law of Sines (Ambiguous Case)
- SAS, SSS: Law of Cosines

**Law of Sines**
\[
\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}
\]

**Law of Cosines**
\[
c^2 = a^2 + b^2 - 2ab \cos(C) \\
a^2 = b^2 + c^2 - 2bc \cos(A) \\
b^2 = a^2 + c^2 - 2ac \cos(B)
\]
Practice

For the following problems, sketch the triangle and solve for the missing sides and angles:

1. $\angle A = 22^\circ$, $\angle B = 95^\circ$, $a = 420$
2. $\angle A = 50^\circ$, $\angle B = 68^\circ$, $c = 230$
3. $a = 28$, $b = 15$, $\angle A = 110^\circ$
4. $a = 3$, $b = 4$, $\angle C = 53^\circ$
5. $b = 60$, $c = 30$, $\angle A = 70^\circ$
6. $a = 20$, $b = 25$, $c = 22$
Trig Formulas
Trig identities

- Power reduction
- Sum and difference
- Pythagorean Identities

\[ a^2 + b^2 = c^2 \]
Trig identities

Let’s try some together:

1. Write in only first powers of cosine: \( \cos^4(2x) \)

2. Given the following values, calculate: \( \sin(A+B) \)
   \[
   \sin A = \frac{15}{12}, \cos A = \frac{8}{17}, \sin B = \frac{17}{13}, \cos B = \frac{12}{13}
   \]

3. Suppose that \( \tan \theta = x \), where \( \theta \) is in the third quadrant and \( x \) is a positive number. What is \( \sin \theta \)?
Trig identities

Try these yourself:

1.) Show that

\[
\frac{\sin(x) - 1}{\sin(x) + 1} = -\frac{\cos^2(x)}{(\sin(x) + 1)^2}
\]
Solving Trig Equations
Solving Trig Equations

Start working like any other equation.

Substitute using trig formulas if needed.

Finish by evaluating the trig functions.

Remember periodicity!
Solving Trig Equations

Let’s try one together:

Solve:

\[ \log_3(2 \sin(x)) = 0 \]
Solving Trig Equations

Try these for yourself:

Solve for x:

1. \( \sin\left(\frac{x}{2}\right) = \cos\left(\frac{x}{2}\right) \)

2. \( 4 \cos^2 x - 4 \cos x + 1 = 0 \)
Study Tips
## What can you do before the semester starts:

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<th>Mentality</th>
<th>Be proactive</th>
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<td>Explore online resources</td>
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<td>Converse</td>
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<tr>
<td>Locate</td>
<td>Find resources on campus, such as CAPS and tutoring</td>
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<tr>
<td>Study</td>
<td>Form a study group, develop a study plan</td>
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Throughout the semester

GO TO CLASS

STAY ON TOP OF HOMEWORK

GO TO PROFESSOR AND TA OFFICE HOURS, CAPS, CALC TABLE.
**Start Your Semester Off Right**
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**Pre-Semester Prep Workshop Series**

These interactive workshops will review all foundational material leading up to the specified course so you are better equipped to hit the ground running.

**Synchronous in-person in the ESS suite & virtual via Zoom**

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*Attend these sessions & give feedback for access to a general knowledge exam.

RSVP is preferred but not required

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